RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2016

THIRD YEAR [BATCH 2014-17]

Date : 22/12/2016 **MATHEMATICS [Honours]**

Time : 11 am – 3 pm **Paper : VI** Full Marks : 100

[Use a separate Answer Book for each Group]

Group – A

Unit - I

(*Answer any six questions*) [6×5]

1. Explain the term 'Noise Level' in a difference table. Prove that Lagrange's interpolation polynomial over a given set of data is unique. [2+3]

2. Obtain Newton's forward interpolation polynomial (without error term) for the equi-distant nodes $x_0, x_1, ..., x_n$. When the formula may be used? [4+1]

3. Obtain numerical differentiation formula based on Lagrange's interpolation formula at an interpolating point and also at a non-interpolating point. [3+2]

4. Obtain the Composite form of Simpson's $\frac{1}{3}$rd rule. Prove that Simpson's $\frac{1}{3}$rd rule gives exact result if applied to any polynomial of degree 3. [3+2]

5. Describe Gauss-Siedel Iterative method for solving a system of linear equations, mentioning stopping criterion. Also state the condition of convergence. [4+1]

6. Discuss Bisection method to find a simple root of the equation f(x) = 0. Why this method is always convergent? [4+1]

7. Discuss fixed point iteration for finding a simple root of f(x) = 0 stating the condition of convergence. [4+1]

8. Describe the power method to calculate numerically largest eigen value of a real symmetric matrix of order n. [5]

9. Solve by Euler's method the following differential equation for x = 1 by taking h = 0·2 :
$\frac{dy}{dx} = xy$, y = 1 when x = 0. [Obtain the result correct to four decimal places] [5]

Unit - II

(*Answer any four questions*) [4×5]

10. A particle moves along the curve :
$x = t^3 + 1,\ y = t^2,\ z = 2t + 5$.
Obtain the components of its velocity and acceleration at t = 1 in the direction of $\hat{i} + \hat{j} + 3\hat{k}$. [5]

11. Find the constants p, q such that the surfaces :
$px^2 - qyz = (p+2)x$ & $4x^2y + z^3 = 4$ cut orthogonally at the point (1, –1,2). [5]

12. If $\vec{a}$ is a constant vector, then prove that (i) Curl $(\vec{a}.\vec{r})\vec{a} = \vec{0}$, (ii) Curl $\left(\frac{\vec{a} \times \vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{a} \cdot \vec{r})$, where $r = |\vec{r}|$. [5]

13. Define circulation of a continuous vector field $\vec{F}$ around a closed curve C in a region R. Hence state the condition for $\vec{F}$ to be irrotational. Use Stoke's theorem to show that $\vec{F}=y^2\hat{i}+x^2\hat{j}-(x+z)\hat{k}$ is irrotational in the triangular region (0, 0, 0), (1, 0, 0), (0, 1, 0). [1+1+3]

14. Verify Green's theorem in the plane for $\oint_C \{(xy+y^2)dx+x^2dy\}$ where C is the closed curve of the region bounded by y = x and y = x^2. [5]

15. Use divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,dS$, where $\vec{F}=3xz\hat{i}+y^2\hat{j}-3xz\hat{k}$ and S is the surface of the cube bounded by the planes x = 0, y = 0, z = 0, x = 1, y = 1, z = 1. [5]

Group – B

Unit - III

(*Answer any three questions*) [3×10]

16. a) Obtain the conditions for a given straight line to be a principal axis of a material system at some point of its length. If so, find the other two principal axes. [6]

b) If a rigid body swing under gravity about a fixed horizontal axis, show that the time of complete oscillation is $2\pi\sqrt{\frac{k^2}{gh}}$, where k is the radius of gyration about the fixed axis and h is the distance between the fixed axis and the centre of inertia of the body. [4]

17. a) Three uniform rods AB, BC, CD are hinged freely at their ends, B and C, so as to form three sides of a square and are laid on a smooth horizontal table. The rod AB is struck at its end A by a horizontal blow P which is perpendicular to its length. Show that the initial velocity of A is 19 times that of D, and that the impulsive actions at B and C are $\frac{5}{12}P$ and $\frac{1}{12}P$ respectively. [7]

b) A uniform rod of length 2a, is with one end in contact with a smooth horizontal table and is then allowed to fall. If α be its initial inclination to the vertical, show that its angular velocity when it is inclined at an angle θ is $\left\{\frac{6g}{a}\cdot\frac{\cos\alpha-\cos\theta}{1+3\sin^2\theta}\right\}^{\frac{1}{2}}$. [3]

18. a) A uniform solid cylinder is placed with its axis horizontal on a plane whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane for pure rolling is $\frac{1}{3}\tan\alpha$. [6]

b) State and prove the Principle of Conservation of energy. [4]

19. a) A circular homogeneous plate is projected up a rough inclined plane with velocity V without rotation, the plane of the plate being in the plane of greatest slope. Show that the plate stops sliding after a time $\frac{V}{g(3\mu\cos\alpha+\sin\alpha)}$, where μ is the coefficient of friction and α is the inclination of the inclined plane. [5]

b) A thin rod of length 2a revolves with uniform angular velocity ω about a vertical axis through a small joint at one extremity of the rod, so that it describes a cone of semi-vertical angle α. Show that $\omega^2=\frac{3g}{4a\cos\alpha}$. [5]

20. a) Show that the angular momentum of a rigid body moving in two dimensions about the origin is $Mvp+Mk^2\dot{\theta}$, where the notations have their usual significance. [5]

b) An elliptic lamina of eccentricity e is rotating with angular velocity ω about one latus rectum; suddenly this latus rectum is set free and the other is fixed. Show that the new angular velocity is $\omega\dfrac{(1-4e^2)}{(1+4e^2)}$. [5]

Unit – IV

(*Answer any two questions*) [2×10]

21. a) A comet is moving in a parabolic orbit about the Sun as focus; when at one end of the latus rectum its velocity suddenly becomes altered in the ratio n : 1, where n < 1; show that the comet will describe an ellipse whose eccentricity is $\sqrt{1-2n^2+2n^4}$. [5]

b) The volume of a spherical raindrop falling freely increases at each instant by an amount equal to λ times its surface area at that instant. If the initial radius of the raindrop be a, find the velocity at the end of time t and the distance fallen through in that time. [5]

22. a) A planet of mass M and periodic time T, when at its greatest distance from the Sun comes into collision with a meteor of mass m, moving in the same orbit in the opposite direction with velocity v; if $\dfrac{m}{M}$ be small, show that the major axis of the planet's path is reduced by $4\dfrac{m}{M}\cdot\dfrac{vT}{\pi}\sqrt{\dfrac{1-e}{1+e}}$. [6]

b) Establish the formula $F(t)=m(t)\dfrac{dv}{dt}+\lambda V$ for the motion of a particle of varying mass m(t) with velocity v under a force F(t), matter being emitted at a constant rate λ with velocity V relative to the particle. [4]

23. a) A particle is projected along the inner surface of a rough sphere and is acted on by no forces. Show that it will return to the point of projection at the end of time $\dfrac{a}{\mu V}\left(e^{2\pi\mu}-1\right)$, where a is the radius of the sphere, V is the velocity of projection and μ is the coefficient of friction. [5]

b) A particle describes an ellipse of eccentricity e about a centre of force at a focus. When the particle is at one end of a minor axis, its velocity is doubled. Prove that the new path is a hyperbola of eccentricity $\sqrt{9-8e^2}$. [5]

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